Calculated Dose Rates in Jupiter's Van Allen Belts

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A series of particle flux and dose rate calculations have been carried out based upon the assumption that the decimetric rf noise emanating from the vicinity of Jupiter is synchrotron radiation from electrons trapped in a dipole magnetic field. Based upon plasma stability considerations and observations in the Earth's Van Allen belts, a relationship between the surface equatorial magnetic field strength (H_0), the spatial electron density (N) and the effective inner radius (a) of the belts was obtained. Three self-consistent belt models were considered (H_0 = 2, 5, and 15 oe) and the point tissue dose rates behind 0.1-10 g/cm² aluminum were calculated. Based upon these assumptions, these dose rates were approximately constant (± a factor of 3) out to ~15 Jupiter radii, whether the Jovian belts consisted of a large number of low-energy particles (small H_0) or a smaller number of higher-energy particles (large H_0).

Nomenclature

a = effective inner radius of Van Allen belt
B = magnetic flux density
C = flux-tissue dose conversion function
c = free space velocity of light
DR = dose rate
dv = volume element
E = particle energy
E_0 = characteristic particle energy for exponential spectra
E'' = shield cutoff energy
F = rf frequency differential power spectrum
H = magnetic field strength
$H_\perp$ = magnetic field strength component perpendicular to particle motion
H_0 = magnetic field strength at planetary equator
L = McIlwain parameter (~planetary radius at magnetic equator)
m_0 = electron rest mass
N = particle density relative to plasma stability limit
n = constant from $R = \delta E^n$ fit to range-energy relation
P_e = total rf radiated power per electron
P_{total} = total rf radiated power from Van Allen belts
R = particle range in matter
R_E = equatorial radius of the Earth
R_J = equatorial radius of the planet Jupiter
r = radius (polar coordinate)
X = shield thickness
Z = atomic number of shield material
α = pitch angle for Van Allen belt particles
γ = ratio of moving to rest electron masses
δ = constant from $R = \delta E^n$ fit to range-energy relation
θ = angle (polar coordinate)
ν = rf frequency
ν_B = cyclotron resonance frequency
ν_c = characteristic synchrotron frequency
ν_p = plasma resonance frequency
ρ = particle density
ρ_e = electron density
ϕ = particle flux

I. Introduction

THE planet Jupiter is one of the most interesting within our solar system. Based upon the current development trends in boosters and spacecraft, it is probable that sometime in the next decade an unmanned spacecraft will be launched toward Jupiter. In order to design and fabricate such a spacecraft it is necessary to know as much as possible about the nuclear radiation environment in the vicinity of Jupiter.

It is currently believed that Jupiter is surrounded by energetic particles trapped in the magnetic field of that planet (Van Allen belts).[1-3]

The evidence for the existence of such belts consists of two types of nonthermal radio frequency noise received from the vicinity of Jupiter. The decametric noise ($\nu \sim$ 10–60 MHz) consists of sporadic bursts[4-10] whose occurrence has been definitely correlated with the orbital motion of the Jovian moon Io (orbit radius ~ 5.9 Jupiter radii).[11-13] The measurements also show a possible correlation with the satellite Europa (orbit radius ~ $15R_J$) as well.[14] These decametric noise bursts usually only last for a few seconds, but exhibit energy fluxes up to 10^{-20} w/m² at the Earth (inferior conjunction).[15,16] The decimetric noise ($\nu \sim$ 300–5000 MHz) consists of a quasi-steady-state emission[17,18] largely emanating from a region having approximately the same polar dimension but three times the axial dimension as the planet.[19,20] This decimeter noise energy flux as measured at inferior conjunction with Earth is shown in Fig. 1.

The source(s) of these nonthermal radio noises has been the object of considerable study. Although the origin of the decametric radio bursts is far from settled,[21-24] there is reasonable general agreement that the decimetric radiations are

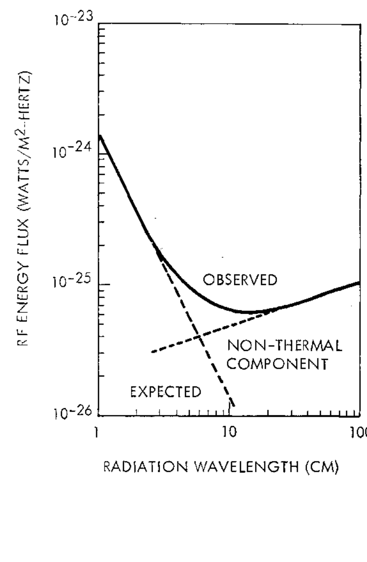

Fig. 1 rf decimetric noise received from Jupiter at inferior conjunction (Earth-Jupiter separation = 4.1 a.u.).

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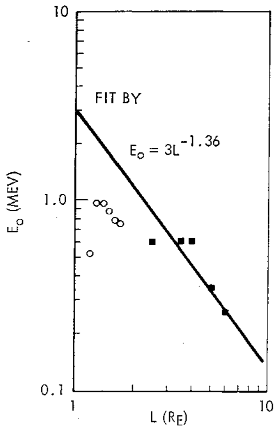

Fig. 2 Radial dependence of E_0 in the Earth's Van Allen belt, based upon an $e^{-E/E}$ energy spectrum.

due to synchrotron emission from relativistic electrons trapped in the Jovian magnetic field.[25–27] The magnetic field strength requirements are less for this mechanism than for any reasonable proposed alternative (e.g., cyclotron radiation)[28,29] and similar rf noise was observed emanating from the high-energy electrons injected into the Earth's Van Allen belt by the Starfish high-altitude nuclear detonation of 1962.[30] Polarization measurements are consistent with this explanation also.[31] There is less agreement concerning the source of the decametric rf bursts. Polarization measurements[32,33] provide clues but no favored explanation. These rf bursts provide one of the incentives for sending a spacecraft to the vicinity of Jupiter, as well as one of the environments that must be taken into account in the design of the spacecraft communications systems.

The object of this study was to calculate the particle fluxes and dose rates expected in the Jupiter Van Allen belts, based upon the most probable available information. The assumptions incorporated include: 1) Jupiter has a dipole magnetic field; 2) the decimetric radio noise received from Jupiter is due to synchrotron emission from electrons trapped in that dipole magnetic field; and 3) the trapped particle characteristics (limiting fluxes, energy spectra, etc.) at any point in Jupiter's magnetic field are the same as those observed in the Earth's Van Allen belts at the same field strength.

In the following sections the calculations of the Jovian Van Allen belt characteristics are presented, based upon these assumptions.

II. Calculation of Jupiter's Magnetic Field Strength

The characteristics of the Jovian Van Allen belts all depend upon the strength of Jupiter's magnetic dipole. It is possible to calculate the minimum field strength at the magnetic equator (H_0) by considering the plasma stability limits for magnetically trapped particles. The logic is that the stronger the magnetic field, the smaller the number of particles required to account for the observed intensity of the decimetric (assumed to be synchrotron) radiation. Conversely, as the value of H_0 is reduced while the required number of trapped particles increases, the number which can be stably contained decreases. For some value of H_0, these two particle populations are equal, i.e., the magnetic field can barely hold all the particles (electrons) necessary to account for the observed decimetric radiation intensity. Although there is no assurance that the particle populations in Jupiter's Van Allen belts approach this stability limit, the calculation of the minimum value of H_0 on the assumption that they do, provides a convenient starting point.

According to Kennel and Petschek[34] the limiting fluxes at the Earth above a nominal energy (40 kev) are approximately

$$\phi(>40 \text{ kev}) = (7 \times 10^{10}/L^4)\text{electrons/cm}^2 - \text{sec} \quad (1)$$

where L (McIlwain parameter) is in Earth radii. The limiting flux is a function of local time, but this effect (small for $L \leq 3$) is neglected here. It will be noted that while the Earth's electron fluxes have been observed to approach this limit, the time-averaged observations as tabulated by Vette[35] are approximately an order of magnitude lower.

The field strength (H_0) at the surface of the Earth at the geomagnetic equator is $\sim$0.3 oe ($B_0 \sim 0.3$ gauss). Since the field strength of a dipole varies inversely with the cube of the distance, it is possible to generalize Eq. (1) by writing

$$\phi(>40 \text{ kev}) = (3.5 \times 10^{11}H_0^{4/3}/L^4) \text{ electrons/cm}^2 \quad (2)$$

It will be noted that this expression retains the same limiting flux for the same magnetic field strength at any L value, and it reverts to Eq. (1) for $H_0 = 0.3$ oe.

The energy spectrum for electrons trapped in the Earth's Van Allen belt may be fitted by an expression of the form

$$\phi(>E) = \phi_0 e^{-E/E_0} \quad (3)$$

where E and E_0 are energies (Mev). Based upon fits to the data as tabulated by Vette, the quantity E_0 may be represented by (see Fig. 2)

$$E_0 = (3/L^{1.36})\text{Mev} \quad (4)$$

As before, it is possible to generalize this expression by writing

$$E_0 = (5.25H_0^{0.455}/L^{1.36})\text{Mev} \quad (5)$$

It will be noted that Eq. (5) has the correct dependence on H_0 and reverts to Eq. (4) for $H_0 = 0.3$ oe.

By combining Eqs. (2) and (5) the flux for any L value in a dipole magnetic field whose strength at the surface of the planet is H_0 may be written

$$\phi(>E) = \frac{3.5 \times 10^{11}H_0^{4/3}}{L^4} e^{-E/E_0} \frac{\text{electrons}}{\text{cm}^2 - \text{sec}} \quad (6)$$

where

$$E_0 = 5.25H_0^{4.455}/L^{1.36} \text{ Mev}$$

This expression yields essentially identical fluxes for $E = 40$ kev (0.04 Mev) and $E = 0$. Any error so introduced will be small, since synchrotron radiation is due to relativistic electrons ($E \gtrsim 0.5$ Mev) and the fit to the energy spectra [Eq. (3)] is most accurate for $E \geq 0.5$ Mev. The fact that the electrons of interest are relativistic facilitates the valid approximation

$$\rho(>E) \sim \frac{\phi(>E)}{c} \frac{\text{electrons}}{\text{cm}^3}, \ \rho(E) \doteq \frac{\phi(E)}{c} = \frac{2.23H_0^{0.876}}{L^{2.64}} \cdot \exp\,(0.19L^{1.36}E/H_0^{0.455}) \frac{\text{electrons}}{\text{cm}^3 - \text{Mev}} \quad (7)$$

where c is the velocity of light.

It will be noted that due to the generalization of the equations obtained from fits to the characteristics of the Earth's Van Allen belts, they have become applicable to any planet with a dipole magnetic field. It is only necessary to insert the equatorial (magnetic) field strength and the L value in planetary radii to obtain the limiting electron flux or density.

The frequency spectrum of synchrotron radiation emitted by a single relativistic electron integrated over all angles[36] is shown in Fig. 3. This spectrum may be approximated by the expression

$$F(\nu,H) = 4.12 \times 10^{-29}(\nu/\nu_c)^{0.29} e^{-(\nu/\nu_c)}\text{W/Hz} \quad (8)$$

where

$$\nu_c = 1.58 \times 10^7 H_\perp E^2 \text{Hz}$$

In these expressions $H_\perp$ is in oe and E is the electron energy in Mev. Integrating this expression over all frequencies yields as the total power radiated by a single relativistic electron

$$P_e = 5.86 \times 10^{-22}(H_\perp \cdot E)^2 \tag{9}$$

The subscript $\perp$ on H refers to the fact that it is only the component of the magnetic field normal to the electron trajectory which is effective in producing synchrotron radiation. The difference between $H_\perp$ and H is small at the magnetic equator and vanishes at the mirror points. The first invariant of a magnetically trapped particle's motion requires that

$$H/\sin^2\alpha = \text{const} \tag{10}$$

Equatorial pitch angle distributions measured in the Earth's Van Allen belts are sharply peaked around 90°, and there are reasons to believe Jupiter's may be even more so. According to Gledhill,[37] Piddington,[38] and others, if the Jovian magnetosphere corotates with the planet, the centrifugal force will tend to flatten Jupiter's Van Allen belts considerably. The spatial dimensions of the decimetric radiation sources (~1 planetary diameter in the polar direction, ~3 or more planetary diameters in the equatorial direction) tend to confirm this expectation. Since any flattening of the Jovian Van Allen belts must be accompanied by a sharply limited equatorial pitch angle distribution, it may be assumed that the normal component of the magnetic field and the total field are essentially the same.

The total decimetric radiation emitted by the maximum possible number of trapped electrons in the Jovian magnetic field is

$$P_{\text{total}} = \iint P_e(H,E) \cdot \rho_e(E,L) dE\, dV \text{ w} \tag{11}$$

where P_e is given by Eq. (9), ρ_e by Eq. (7), and the integral is to be evaluated over all energies and over the volume of the Jovian Van Allen belts. First, it is necessary to rewrite P_e as

$$P_e = 5.86 \times 10^{-22}[(H_0/L^3)E]^2 \text{ w/electron} \tag{12}$$

Next, it is necessary to estimate the spatial extent of the Jovian Van Allen belt. The magnetic flux lines from a dipole obey the relationship

$$r = L \cos^2\theta \tag{13}$$

where r and θ are polar coordinates. Thus, the spatial extent of the Earth's Van Allen belts in an Earth-centered polar coordinate system is approximately

$$r = a \text{ to } r = R \cos^2\theta$$

where a = effective inner radius = $1.2R_e$ (Earth radii) and R = effective outer radius = $10 \pm 2R_e$ (sunward). For Jupiter, the assumed corresponding spatial extent is

$$r = a \text{ to } r = R \cos^n\theta$$

where $R = 21H_0^{1/3}$ planetary radii and a is in planetary radii also. This value of R was based upon an extrapolated solar wind energy density of 2.5×10^{-4} Mev/cm^3 at Jupiter's mean orbital radius (5.2 a.u.).

Two other assumptions must be mentioned. The integral shown in Eq. (11) assumes that the planet is transparent, which it is not. However, after-the-fact calculations show that this effect is small. Even if the effective location of the radiating electrons is at 1.25 planetary radii, the planet screens only 14% of the radiation. The second assumption is that the synchrotron radiation from the assembly of electrons is simply the sum of that due to each individual electron. According to Ramaty,[39] this is not true unless the cyclotron frequency ν_B is at least comparable to the plasma frequency ν_p. These quantities may be calculated using the equations

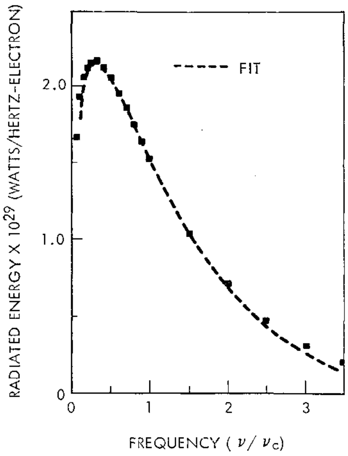

Fig. 3 Spectrum of synchrotron radiation emitted by a single relativistic electron in uniform magnetic field.

$$\nu_B = (eB/2\pi mc)\text{Hz}, \; \nu_p = e(\rho/\pi m)^{1/2}\text{Hz} \tag{14}$$

where e is the electron charge, m the electron mass, and ρ the electron density. The ratio of these quantities may be written

$$\nu_p/\nu_B = 2c(\pi m_0 \gamma \rho)^{1/2}/B \tag{15}$$

where $m_0 = m/\gamma$ = the electron rest mass. Since

$$\begin{aligned} \rho &\sim 11.7B^{4/3} \text{ electrons/cm}^3 \\ \nu_p/\nu_B &\sim 6.5 \times 10^{-2}\gamma^{1/2}/B^{1/3} \end{aligned} \tag{16}$$

Thus, for reasonable values of $\gamma(\leq 10^2)$ and $B(\lesssim 10)$, it is seen that ν_p/ν_B is < 1. For this situation the self-attenuation of synchrotron radiation by the plasma producing it may be neglected. This low plasma density also precludes appreciable spatial distortion of the magnetic field due to the "frozen-in" effect.

The total decimetric radiation emitted by the Jovian Van Allen belts may now be written

$$P_{\text{total}} = \int 5.86 \times 10^{-22} \left(\frac{H_0 E}{L^3}\right)^2 \cdot \frac{2.23 H_0^{0.876} e^{-E/E_0}}{L^{2.64}} dE\, dv \tag{17}$$

Integrating over E from 0 to ∞ yields

$$P_{\text{total}} = 3.78 \times 10^{-19} \int (H_0^{4.24}/L^{12.72}) dv \text{ w} \tag{18}$$

The theorem of Pappus is used to evaluate the integral over the volume of Jupiter's Van Allen belts. This theorem states that the integral of any analytical function over a volume of revolution is the product of the integral over the cross section of the volume and the distance the centroid of this cross-sectional area travels in sweeping out the volume. Employing this theorem and replacing L by r (a fairly good approximation for small L values where the radiation source density is largest, as θ cannot exceed 1~ rad) yields

$$\text{P}_{\text{total}} = 3.78 \times 10^{-19} H_0^{4.24} \int_0^{\theta\max} \int_a^{R\cos^n\theta} \times \frac{r\, dr\, d\theta}{r^{12.72}} \approx 3.6 \times 10^{-20} \frac{H_0^{4.24}}{a^{10.72}} \frac{\text{w}}{\text{cm}} \tag{19}$$

This represents the synchrotron energy radiated from a cross section of Jupiter's Van Allen belts ($\theta_{\max} \sim 1$ rad). Since the centroid of this cross section is ~1.1a, the total radiated power is

$$P_{\text{total}} = 2.5 \times 10^{-19} \frac{H_0^{4.24}}{a^{9.72}} \text{ w} \tag{20}$$

There is a basic inconsistency in units here. The electron

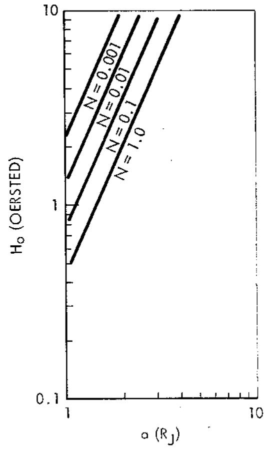

Fig. 4 Calculated relationship between H_0, N, and a for Jupiter's Van Allen electron belt where H is the equatorial magnetic field strength at surface of planet, N is the electron population relative to that at calculated plasma stability limit, and a is the effective inner radius of Van Allen electron belt in planetary radii.

densities used were in cm^{-3}, but the spatial dimensions of the Van Allen belts were in units of planetary radii. For Jupiter, the radius is $\sim 7.2 \times 10^9$ cm, so the coefficient of the preceding equation must be multiplied by the cube of this number. The result is

$$P_{\text{total}} = 9.3 \times 10^{+10} \frac{H_0{}^{4.24}}{a^{9.72}} \text{ w} \tag{21}$$

The decimetric rf noise observed at inferior conjunction with the Earth (separation distance 4.1 a.u.) may be represented by the expression (see Fig. 1)

$$\begin{aligned} P(\nu) &= 2.3 \times 10^{-26} \cdot \lambda^{1/3} \\ &= 7.15 \times 10^{-23} \cdot \nu^{-1/3} \frac{\text{w}}{\text{m}^2 - \text{Hz}} \end{aligned} \tag{22}$$

Integrating this $\nu = 0(\lambda = \infty)$ to $\nu = 1.2 \times 10^{10}$ ($\lambda = 2.5$ cm) yields 5.6×10^{-16} w/m^2 at Earth. Assuming isotropy over a sphere of radius 4.1 a.u. to obtain the total synchrotron power radiated from Jupiter, the result is 2.67×10^9 watts. Equating this to the result of the integration just carried out [Eq. (12)] leads to

$$a^{9.72}/H_0{}^{42.4} \leq 35 \text{ or } H_0 \geq 0.44a^{2.29} \text{ oe} \tag{23}$$

This is the calculated requirement for the minimum equatorial magnetic field strength for the planet Jupiter. It presumes that the belts are 100% populated. If the relative electron population is $N(N \leq 1)$, the required minimum field is increased. Specifically, the total radiated power becomes, assuming that the electron population is decreased uniformly at all energies and at all positions,

$$P_{\text{total}} \approx 9.3 \times 10^{10} (\text{N} \cdot H_0{}^{4.24}/a^{9.2}) \text{w}$$

Proceeding as before leads to the generalized relationship

$$H_0 \geq (0.^{44}a^{2.29}/N^{0.236}) \text{ oe} \tag{24}$$

This function is shown in Fig. 4. As expected, the minimum equatorial field increases rapidly as the effective inner radius of the Jovian Van Allen belts increases. A reasonable minimum value of a is $\sim 1.2R_J$, analogous to the effective inner radius of the Earth's Van Allen belts. A probable upper limit for a is $\sim 2R_J$. Although it was thought at one time that Jupiter's magnetic field was highly eccentric,[40,41] it is difficult to imagine such a rapidly spinning body having its heavy materials (including those responsible for the magnetic field) appreciably off center. The center of mass of the planet lies essentially at its geometric center. Therefore, values of a between 1.2 R_J and 1.5 R_J are most probable.

The probable value of N (the relative electron population) is much more uncertain. The sources of the particles trapped in the Earth's Van Allen belt are by no means well understood. The sources of the particles trapped in Jupiter's Van Allen belts are, therefore, speculative. If albedo neutrons resulting from galactic (cosmic) particles are an important source, the Jovian belts may be fairly well populated in spite of the low mass of the atoms making up Jupiter's atmosphere. If particles from the sun are important sources of Van Allen particles, the fact that Jupiter is at ~ 5.2 a.u. will act to limit N. In any event, the intensity of the decametric radio noise suggests that appreciable numbers of particles are violently disturbed by Jupiter's natural satellites. Thus, it is probable that N is less than 0.1, possibly an order of magnitude or more less. Knowledge of the magnetospheric limit for Jupiter would remove most of this uncertainty.

A simple consistency check is in order here. Having deduced that the equatorial magnetic field at the surface of Jupiter is on the order of a few oersteds, the electron energy at 1.2 – 1.5 R_J must be (based upon the peak of the synchrotron radiation function)

$$E_e(\text{Mev}) = 4.66 \times 10^{-4} (\nu/H_\perp)^{1/2}$$

where ν is in hertz and $H_\perp$ is in oersteds. For the Jovian belts, $\nu \approx 10^9 \pm$ a factor of 3. Thus, E_e will be on the order of 10 Mev. The Jovian belt electron energy distribution was assumed to be of the form (based upon the measured spectra in the Earth's Van Allen belts)

$$\phi(>E,r) = \phi_0/L^4 \cdot \exp -(L^{1.36}E/3) \text{electrons}/$$
$$(\text{cm}^2 - \text{sec}) \sim \exp -0.41E \text{ at } r \sim 1.2R_J$$

where E is in Mev.

The flux of electrons whose energies are >10 Mev is thus 1.66% of that >0 Mev at this radius. However, this calculation was based upon the peak of the synchrotron radiation function, and the electron emits over half of its energy at frequencies >4 times the peak frequency. The electron energy requirements thus are more like 5 Mev, and $\sim$13% of the electrons have energies $\geq$5 Mev at 1.2 R_J according to our model. Thus, the model used for these calculations not only accounts for the rf power received, but also yields rf frequencies in agreement with those observed.

For the purpose of these calculations, the following values of H_0 were considered:

$$H_0 = 2 \text{ oe}(N = 0.01, a = 1.2 R_J)$$
$$H_0 = 5 \text{ oe } (N = 0.001, a = 1.4 R_J)$$
$$H_0 = 15 \text{ oe } (N = 0.0001, a = 1.8 R_J)$$

Flux and dose rate calculations for these three cases are carried out in the next section.

III. Calculations of Dose Rates in Jupiter's Van Allen Belts

The equation for the electron flux as a function of H_0 (magnetic field strength at the visible surface of the planet) has already been presented (Eq. 6). Incorporating the dependence on N (the relative particle population) is straightforward. The result is

$$\phi(>E) = \frac{3.5 \times 10^{11} N H_0{}^{4/3}}{L^4} e^{-E/E_0} \frac{\text{electrons}}{\text{cm}^2 - \text{sec}} \tag{25}$$

where

$$E_0 = (5.25 H_0{}^{0.455}/L^{1.36}) \text{Mev}$$

At the magnetic equator $L = r$. By limiting the calculations to the dose rates in the plane of the Jovian magnetic equator the situation is simplified. From this point on r is substituted for L.

It is interesting, in passing, to compare the measured electron fluxes in the Earth's Van Allen belt with those obtained from Eq. (25). The time-averaged flux data, as compiled by Vette,[35] is shown in Fig. 5 along with the fluxes calculated using $N = 0.1$ and $H_0 = 0.3$. It is apparent that whereas the agreement if fairly good for $r \geq 5R_e$ and fair for $r \leq 1.5\ R_e$ at the lower energies, it is poor for $1.5\ R_e < r < 5R_e$. Obviously, N, the particle population related to saturation, is a function of L, with N varying between $\sim 10^{-4}$ and $\sim 10^{-1}$. Therefore, these values of N may be considered as reasonable for calculating dose rates in Jupiter's Van Allen belts.

Fortunately, the electron flux to dose conversion function is quite flat, having the approximate value $C = 3 \times 10^{-8}$ rad-cm^2/electron.[42] This is within $\pm 20\%$ of the Halpern and Hall calculations for $E = 0.5$ Mev to $E = 10$ Mev. Therefore, to calculate the point (meatball) dose due to electrons in the Jovian Van Allen belts, it is only necessary to multiply the integral flux above the shield cutoff energy (E'') by the flux to dose conversion function (C) to obtain

$$DR(X,r) = \frac{3.8 \cdot 10^7 N H_0^{4/3} e^{-E'/E_0}}{r^4} \frac{\text{rads}}{\text{hr}} \tag{26}$$

For the shield thickness X(g/cm^2) it is possible to write

$$E' = \frac{2.8\ X^{0.9+0.15 \log X}}{Z^{0.2}}\ \text{Mev} \tag{27}$$

where Z is the atomic number of the shield material. The dose rates for 0.1, 1.0, and 10 g/cm^2 aluminum ($E' = 0.29$, 1.7, and 19 Mev, respectively) have been calculated. It is seen that for close-in points, large values of N (and their correspondingly small values of H_0) produce the larger dose rates. This is due to the fact that N varies much more rapidly than $H_0^{4/3}$, and thus its effect is overshadowing. At appreciable distances from Jupiter, large values of H (and the correspondingly small values of N) produce the larger dose rates. Beyond the magnetospheric limit ($\sim 21 \cdot H_0^{1/3}$ planetary radii), of course, the Van Allen fluxes and dose rates drop to zero.

The calculated dose rates do not vary nearly as much as the choice of input parameters (H_0,N,a) would suggest. In Fig. 6 the envelopes of the calculated electron dose rates in the plane of Jupiter's magnetic equator are shown. Considering that the equatorial magnetic field strength has a probable uncertainty approaching an order of magnitude, the dose rate calculations yield remarkably consistent results. Thus, whether Jupiter's electron belts are composed of a large number of particles having a relative soft energy spectrum ($H_0 \sim 2$ oe) or a smaller number of particles having a hard energy spectrum ($H_0 \sim 15$ oe) the dose rates out to $\sim 15\ R_J$ are approximately the same.

The estimation of the proton fluxes and dose rates is not quite straightforward. According to Kennel and Petschek[34] the limiting proton and electron fluxes are equal and the limiting differential spectrum is flat if the plasma frequency is appreciably less than the cyclotron frequency. Measurements in the Earth's Van Allen belts do not confirm these

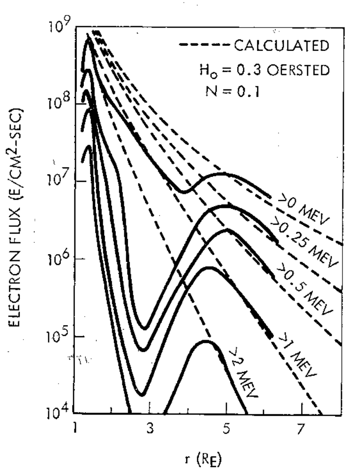

Fig. 5 Electron fluxes in the Earth's Van Allen belt at the geomagnetic equator.

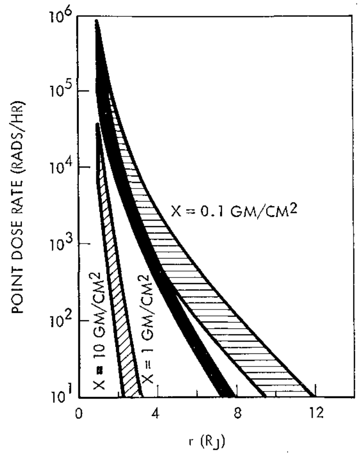

Fig. 6 Envelope of the calculated electron dose rates in the plane of Jupiter's magnetic equator.

conclusions. Therefore, while the plasma stability limits are apparently valid for estimating the electron flux above 40 kev, direct measurements of the earth's Van Allen belts are preferable for estimating proton fluxes. The time-averaged proton energy spectra in the Earth's Van Allen belt have been fit by McIlwain and Pizzella (31–43 Mev)[43] and by Imhof and Smith (59–148 Mev)[44] with expressions of the form

$$\phi(E) \sim \text{const}\ e^{-E/E_0} \frac{\text{protons}}{\text{cm}^2 - \text{sec} - \text{Mev}} \tag{28}$$

where $E_0 \sim 400\ L^{-5}$.

Since E_0 varies inversely with the fifth power of L and the magnetic field strength varies (at the equator) inversely with the cube of L, it is possible to generalize this equation to

$$E_0 = (3000\ H_0^{5/3}/L^5)\ \text{Mev} \tag{29}$$

where H_0 is the equatorial magnetic field at the planetary surface in oersteds.

Based upon a theoretical L^{-4} dependence and the time-averaged proton fluxes as compiled by Vette,[35] the integral energy spectra in the Earth's Van Allen belts are approximately

$$\phi(>E) = \frac{2.5 \times 10^7 e^{-E/E_0}}{L^4} \frac{\text{protons}}{\text{cm}^2 - \text{sec}} \tag{30}$$

This expression yields fluxes approximately an order of magnitude higher than those observed at $E > 30$ Mev, consistent with the time-averaged electron fluxes, which are at $\sim 10\%$ their stability limit. Below ~ 5 Mev the expression yields fluxes lower than those measured. However, the fits to the energy spectra were made at high energies (31–148 Mev) and the important dose-rate-producing protons (1–10 g/cm^2 shielding) lie in this energy region. Generalizing Eq. (30) as before, to make it applicable to a planet with an arbitrary surface equatorial magnetic field strength H_0, the result is

$$\phi(>E) = \frac{1.25 \times 10^9 N H_0^{4/3} e^{-E/E_0}}{L^4} \frac{\text{protons}}{\text{cm}^2 - \text{sec}} \tag{31}$$

where $E_0 = (3000\ H_0^{5/3}/L^5)$Mev.

The insertion of N (the relative particle population, with $N = 1$ representing saturation) is to be noted. From this point on, Eq. (31) is considered to be the companion of Eq. (25), the limiting electron flux, and N is assumed to have the same value in both equations.

The comparison of the proton fluxes calculated using Eq. (31) with the time-averaged measurements as compiled by Vette is shown in Fig. 7. The situation is similar to that for electrons—reasonable agreement for the input values chosen ($H_0 = 0.3$, $N = 0.1$) for larger values of $r(\geq 2.5R_e)$ and poor agreement for smaller values of r. Again, this is

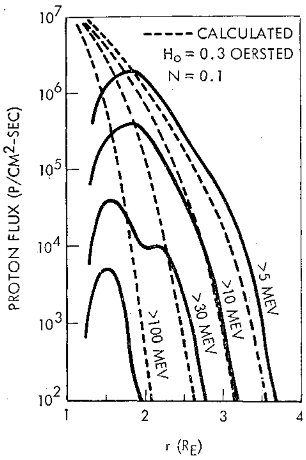

Fig. 7 Proton fluxes in the Earth's Van Allen belt at the geomagnetic equator.

attributed to physical mechanisms which limit N for small values of r.

The calculation of the proton dose rates from the proton fluxes is less straightforward than was the case for electrons. While the electron flux to dose conversion function was approximately a constant, the proton conversion function $C(E)$ may be represented by ($E \leq 1$ Bev)[42]

$$C(E) = 4 \times 10^{-6} E^{-0.8} \frac{\text{rad} - \text{cm}^2}{\text{proton}} \tag{32}$$

The situation is complicated by the fact that the incident proton energy (E) and the energy (E') the proton has after transversing a thickness (X) of shielding are related by an expression of the form

$$E' = (E^n - X/\delta)^{1/n} \tag{33}$$

where E and E' are in Mev, X is in g/cm², and n and δ are constant for any given shield material. For power-law spectra of the form

$$\phi(>E) \sim AE^{-\alpha} \text{ protons/(cm}^2 - \text{sec)}$$

the proton dose rate (DR) may be approximately calculated by the expression

$$DR \sim 2C(E'')A\ (E'')^{-\alpha} \text{ rads/hr} \tag{34}$$

where E'' is the shield cutoff energy (Mev) for protons.[42]

This approach works because most of the dose is produced by protons whose energies are just above E''. Lower-energy protons cannot penetrate the shield, and protons much above E'' are less numerous and less effective at producing dose (on a per-particle basis) than those slightly above E''. The success of the approximation for power law spectra suggests that it may be possible to write for exponential spectra,

$$DR \sim 2C(E'') \cdot Ae^{-E''/E_0} \text{ rads/hr} \tag{35}$$

where $E'' = 39X^{0.57}/Z^{0.15}$ and $C(E'')$ is given by Eq. (32). Substituting yields

$$DR \approx \frac{3.6 \cdot 10^7 \cdot N \cdot H_0 \cdot e^{-E''/E_0}}{L^4} \left(\frac{1}{E''}\right)^{0.8} \left(\frac{\text{rads}}{\text{hr}}\right) \tag{36}$$

The major justification for using this equation is that it has the theoretically expected dependence on the parameters involved, and that it yields dose rates per unit proton flux approximately in agreement with those more precisely calculated for the Earth's Van Allen belts ($N \sim 0.1$, $H_0 \approx 0.3$, $E_0 \approx 400$).[42,45]

Proton point dose rates behind aluminum shields in Jupiter's Van Allen belts were calculated using Eq. (36). Equatorial dose points ($L = r$) were chosen for the three cases previously considered for electrons;

$$H_0 = 2(N = 0.01,\ a = 1.2R_J)$$
$$H_0 = 5(N = 0.001,\ a = 1.4R_J)$$
$$H_0 = 15(N = 0.0001,\ a = 1.8R_J)$$

As for the electron dose rate calculations, the proton dose rates vary surprisingly little. The envelopes of the proton dose rate calculations are shown in Fig. 8.

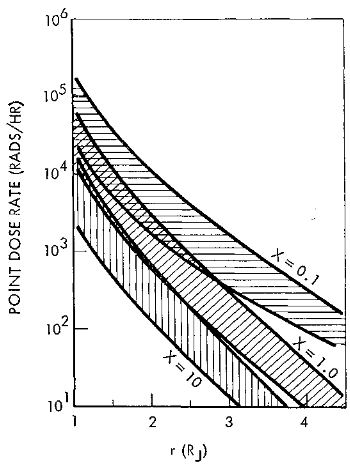

Fig. 8 Envelope of the calculated proton dose rates in the plane of Jupiter's magnetic equator (X in g/cm²).

IV. Conclusions

The calculated equatorial electron and proton dose rates for $r \leq 15\ R_J$ are seen to be relatively insensitive to the parameters selected for Jupiter's Van Allen belts. Beyond $\sim 15R_J$, the extent of the uncertainties associated with these dose rates increases. As the Jovian magnetopause is approached the uncertainties exceed an order of magnitude. In fact, knowledge of the sunward radius of Jupiter's magnetosphere would reduce these flux and dose rate uncertainties considerably.

The assumptions involved in carrying out these calculations must be kept in mind. Although all the fluxes and dose rates are consistent with the characteristics of the decimetric (assumed to be synchrotron) radiation received from Jupiter, there is no guarantee that the radiation is not partially (or totally) generated by some other mechanism. The fact that an explanation is considered to be the most probable one is neither a necessary nor a sufficient condition for its validity.

Nevertheless, it is desirable to estimate as much as possible about an unknown environment before designing, building, and operating instrument-carrying vehicles to probe it. These calculations show that the particle fluxes and dose rates in the outer portions ($r \geq 15R_J$) of Jupiter's magnetosphere should not constitute an unduly severe nuclear radiation environment. However, the inner portions ($r \leq 15R_J$) are most probably populated by electrons and protons whose large fluxes and dose rates should be considered in the design of any Jupiter probe.

References

[1] Roberts, M. S. and Huguenin, G. R., "The Radiation Belt of Jupiter," *Proceedings of the Eleventh International Astrophysical Symposium*, Liege, Belgium, 1962, pp. 569–573.

[2] Roberts, J. A. and Ekers, R. D., "The Position of Jupiter's Van Allen Belt," *Icarus*, Vol. 5, No. 2, March 1966, pp. 149–153.

[3] Hove, L. E. et al., "A Study of Jupiter Flyby Missions, Section 5.3.9 (Radiation Protection)," Rept. FZM-4625, 1966, Convair, Ft. Worth, Texas.

[4] Franklin, K. L. and Burke, B. F., "Radio Observations of the Planet Jupiter," *Journal of Geophysical Research*, Vol. 63, No. 4, Dec. 1958, pp. 807–824.

[5] Kraus, J. D., "Planetary and Solar Radio Emission at 11 Meters Wavelength," *Proceedings of the Institute of Radio Engineers*, Vol. 46, No. 1, Jan. 1958, pp. 266–274.

[6] Douglas, J. N., "A Uniform Statistical Analysis of Jovian Decameter Radiation, 1950–1960," *The Astronomical Journal*, Vol. 65, No. 9, Nov. 1960, pp. 487–488.

[7] Mayer, C. H., "Introductory Review on the Radio Emission of the Planets," Space Research III, *Proceedings of the Third International Space Science Symposium*, Committee on Space Research, 1962, pp. 1006–1020.

[8] Stone, R. G., Alexander, J. K., and Erickson, W. C., "Low Level Decameter Emissions From Jupiter," *The Astrophysical Journal*, Vol. 140, No. 1, July 1964, pp. 374–377.

[9] Carr, T. D. et al., "Spectral Distribution of the Decametric Radiation From Jupiter in 1961," *The Astrophysical Journal*, Vol. 140, No. 2, Aug. 1964, pp. 778–795.

[10] Warwick, J. W. and Kreiss, W. T., "Observations of Jupiter's Sporadic Radio Emission in the Range 7.6-41 Mc/Sec—January 1964 Through May 1965," LGY Solar Activity Report, Series 32, 1964.

[11] Bigg, E. K., "Influence of the Satellite Io on Jupiter's Decametric Emission," *Nature*, Vol. 203, No. 4949, Sept. 1964, pp. 1008–1010.

[12] Alexander, J. K. and Stone, R. G., "Influence of Satellite Io on the 26.3 Mc/Sec Emissions From Jupiter," *The Astrophysical Journal*, Vol. 141, No. 4, May 1965, pp. 1587–1888.

[13] Bigg, E. K., "Periodicities in Jupiter's Decametric Radiation," *Planetary and Space Science*, Vol. 14, No. 8, Aug. 1966, pp. 741–758.

[14] Lebo, G. R., Smith, A. G., and Carr, T. D., "Jupiter's Decametric Emission Correlated With the Longitudes of the First Three Galilean Satellites," *Science*, Vol. 148, No. 3678, June 1965, pp. 1724–1725.

[15] Roberts, J. A., "Radio Emission From the Planets," *Planetary and Space Science*, Vol. 11, No. 2, Feb. 1963, pp. 221–259.

[16] Michaux, C. M., "Handbook of the Physical Properties of the Planet Jupiter," Rept. SP-3031, 1967, NASA.

[17] Miller, A. C. and Gary, B. L., "Measurements of the Decimeter Radiation From Jupiter," *The Astronomical Journal*, Vol. 67, No. 10, Dec. 1962, pp. 727–731.

[18] Rzhiga, O. N. and Trunova, Z. G., "Measurement of Jupiter's Intrinsic Decimeter—Wavelength Radiation," *Soviet Physics—Astronomy*, Vol. 9, No. 1, July-Aug. 1965, pp. 93–95.

[19] Morris, D. and Berge, G. L., "Measurements of the Polarization and Angular Extent of the Decimeter Radiation From Jupiter," *The Astrophysical Journal*, Vol. 136, No. 1, July 1962, pp. 276–282.

[20] McAdam, W. B., "The Extent of the Emission Region on Jupiter at 408 Mc/Sec," *Planetary and Space Science*, Vol. 14, No. 11, Nov. 1966, pp. 1041–1046.

[21] Zhelezniakov, V. V., "On the Theory of the Sporadic Radio Emission From Jupiter," *Soviet Astronomy*, Vol. 2, No. 20, March-April 1958, pp. 206–215.

[22] Chang, D. B., "Amplified Whistlers as the Source of Jupiter's Sporadic Decameter Radiation," *The Astrophysical Journal*, Vol. 138, No. 4, Nov. 1963, pp. 1231–1241.

[23] Smith, A. G. et al., "Occultation Theory of Jovian Radio Outbursts," *Nature*, Vol. 199, No. 4890, July 1963, pp. 267–268.

[24] Duncan, R. A., "Factors Controlling Jovian Decametric Emission," *Planetary and Space Science*, Vol. 14, No. 12, Dec. 1966, pp. 1291–1301.

[25] Chang, D. B. and Davis, L., "Synchrotron Radiation as the Source of Jupiter's Polarized Decimeter Radiation," *The Astrophysical Journal*, Vol. 136, No. 2, Sept. 1962, pp. 567–581.

[26] Six, N. F., "Characteristics and Origin of the Non-Thermal Radio Emission From Jupiter," TN R-60, July 1966, Brown Engineering, Huntsville, Ala.

[27] Korchak, A. A., "Synchrotron Radiation of Charged Particles in a Dipole Magnetic Field," *Soviet Astronomy*, Vol. 7, No. 6, May-June 1964, pp. 764–771.

[28] Field, G. B., "The Source of Radiation From Jupiter at Decimeter Wavelengths-3-Time Dependence of Cylotron Radiation," *Journal of Geophysical Research*, Vol. 66, No. 5, May 1961, pp. 1395–1405.

[29] Ellis, G. R. A., "Radiation From Jupiter at 4.8 Mc/Sec," *Nature*, Vol. 194, No. 4829, May 1962, pp. 667–668.

[30] Nakada, M. P., "Synchrotron Radiation Calculations for the Artificial Radiation Belt," *Journal of Geophysical Research*, Vol. 68, No. 13, July 1963, pp. 4079–4089.

[31] Radhakrishnan, V. and Roberts, J. A., "Polarization and Angular Extent of the 960 Mc/Sec Radiation From Jupiter," *Physical Review Letters*, Vol. 4, No. 10, May 1960, pp. 493–494.

[32] Sherrill, W. M. and Castles, M. P., "Survey of the Polarization of Jovian Radiation at Decameter Wavelengths," *The Astrophysical Journal*, Vol. 138, No. 2, Aug. 1963, pp. 587–598.

[33] Barrow, C. H., "Polarization Observations of Jupiter at Decameter Wavelengths," *Icarus*, Vol. 3, No. 1, May 1964, pp. 66–77.

[34] Kennel, C. F. and Petschek, H. E., "Limit on Stably Trapped Particle Fluxes," *Journal of Geophysical Research*, Vol. 71, No. 1, Jan. 1966, pp. 1–28.

[35] Vette, J. I. et al., "Models of the Trapped Radiation Environment," Vol. I, II, and III, NASA Report SP-3024, 1966 and 1967.

[36] Ginzburg, V. L. and Syrovat-skii, S. I., "Cosmic Magnetic Bremsstrahlung (Synchrotron Radiation)," *Soviet Physics USPEKHI*, Vol. 8, No. 5, March-April 1966, pp. 674–701.

[37] Gledhill, J. A., "Magnetosphere of Jupiter," *Nature*, Vol. 214, No. 5084, April 1967, pp. 155–156.

[38] Piddington, J. H., "Jupiter's Magnetosphere," Rept. 67-63, 1967, Univ. of Iowa, Ames, Iowa.

[39] Ramaty, R., "Influence of the Ionized Medium on Synchrotron Emission of Intermediate Energy Electrons," *Journal of Geophysical Research*, Vol. 73, No. 11, June 1968, pp. 3573–3582.

[40] Warwick, J. W., "Radio Emission From Jupiter," *Annual Review of Astronomy and Astrophysics*, Vol. 2, Annual Reviews, Palo Alto, Calif., 1964, pp. 1–22.

[41] Berge, G. L. and Morris, D., "Decimeter Measurements Relating to the Possible Displacement of Jupiter's Magnetic Dipole," *The Astrophysical Journal*, Vol. 140, No. 3, Oct. 1964, pp. 1330–1332.

[42] Haffner, J. W., *Radiation and Shielding in Space*, Academic Press, New York, 1967.

[43] McIlwain, C. E. and Pizzella, G., "On the Energy Spectrum of Protons Trapped in the Earth's Inner Van Allen Zone," *Journal of Geophysical Research*, Vol. 68, No. 7, April 1963, pp. 1811–1823.

[44] Imhof, W. L. and Smith, R. V., "Proton Intensities and Energy Spectra in the Inner Van Allen Belt," *Journal of Geophysical Research*, Vol. 69, No. 1, Jan. 1964, pp. 91–100.

[45] Keller, F. L. and Pruett, R. G., "The Effect of Charged Particle Environments on Manned Military Space Systems," *Second Symposium on Protection Against Radiation in Space*, NASA Report SP-71, Oct. 1964, pp. 265–286.